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Engineering Notes

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Example Impact of Nonuniform Acceleration Fields on Liquids in Spacecraft

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Nomenclature

a	=	steady acceleration
f	=	liquid fill fraction
M	=	mass of a nearby spacecraft
B	=	Bond number
B_{gg}	=	ratio of gravity-gradient and capillary effects
B_{ng}	=	ratio of nearby-spacecraft and capillary effects
B_{sg}	=	ratio of self-gravitation and capillary effects
G	=	universal gravitational constant, $6.6742 \times 10^{-11}\ \mathrm{m}^3 \cdot \mathrm{kg}^{-1} \cdot \mathrm{s}^{-2}$.
P_a	=	pressure difference due to acceleration
P_c	=	pressure difference due to capillary effects
P_{ng}	=	pressure difference due to a nearby spacecraft
P_{gg}	=	pressure difference due to gravity gradient
P_{sg}	=	pressure difference due to self-gravitation
R_{orb}	=	orbital radius
R_T	=	radius of the spherical tank
Δ	=	separation distance
ρ	=	mass density
σ	=	surface tension
Ω	=	orbital period

I. Introduction

TANK interior geometry and liquid contact angle determine the positioning of liquid propellants contained in spacecraft tanks that are in perfect weightlessness. When the spacecraft accelerates, the mass density and surface tension of the liquid become relevant. The relative magnitude of steady acceleration (or gravity) and capillary forces is described by the Bond number. However, nonzero net acceleration fields exist in orbit even without spacecraft accelerations. The term "net acceleration field" denotes the acceleration environment of locations within the spacecraft. When the spacecraft is in orbit and not maneuvering, the center of mass of the spacecraft travels the orbital path and thus the gravitational attraction and the centripetal acceleration are perfectly balanced at that point in the spacecraft. This condition is commonly called weightlessness or zero gravity. In contrast, points of the spacecraft on the Earth side of the center of mass experience slightly greater gravitational force and slightly lower centripetal acceleration, leading to an imbalance commonly termed the gravity gradient (discussed in numerous dynamics textbooks). It is shown in this paper that even for propellant tanks as small as approximately 1 m radius, it is important to include such nonuniform acceleration fields in the design methods for the spacecraft.

In addition to gravity gradient, a nonuniform net acceleration field in the propellant tank can be caused by the self-gravitation of the liquid propellant and by the close proximity of a massive spacecraft, such as when two spacecraft are docking. Self-gravitation is the normally negligible effect of the gravitational attraction of every liquid molecule on other liquid molecules. Myshkis et al. ([1], p. 6) shows that capillary and self-gravitational effects in spherical liquid masses are equal in magnitude for sphere radius usually under 10 m. Tank diameters on the order of 1 m for satellites are common and larger are anticipated for future space exploration.

Because of the coincidence of sizes for proposed tanks and the length scale of a known nonuniform acceleration field effect, the relative magnitudes of the effects of capillary, steady acceleration, gravity gradient, self-gravitation, and nearby masses should be considered. A scaling analysis is presented, followed by in-depth analysis of the gravity gradient effect for a spherical tank with vane-type propellant management device on the axis. Modeling self-gravitation effects will require additional efforts that include modifications of the computational tool and thus are not yet implemented.

II. Scaling

Scaling of the relative magnitudes of the effects of acceleration, gravity gradient, self-gravitation, and a nearby massive object is quantified by the pressure differences created by each phenomenon. Consider a spherical tank in a satellite in a circular orbit about the Earth. The liquid is described by mass density and the surface tension of the free surface. The mass of the pressurant gas or vapor is negligible. Interface mean curvature is assumed to scale with the inverse of tank radius. Constant factors near unity are omitted from all relations.

The capillary pressure difference across the interface given by the Young–Laplace equation [1] is approximated as

$$P_c \propto \sigma / R_T \tag{1}$$

The affect of constant spacecraft acceleration of magnitude a is [2]

$$P_a \propto \rho a R_T \tag{2}$$

Pressure difference from the center of a liquid sphere to the edge caused by self-gravitation (see [1], p. 6) is

$$P_{sg} \propto \rho^2 G R_T^2 \tag{3}$$

Gravity gradient at orbital radius R_{orb} with orbital period Ω and $R_T \ll R_{orb}$ is approximately

$$P_{gg} \propto \Omega^2 \rho R_T^2 \tag{4}$$

The pressure difference induced by a point or spherical mass M located a distance Δ from the center of the tank, such as another spaceship docking with the host ship of the tank, is

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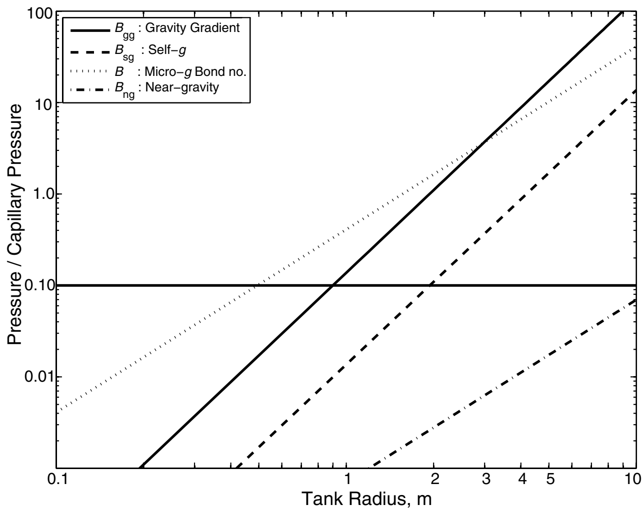

Fig. 1 Scaling results for liquid oxygen in a circular orbit 800 km above the Earth.

$$P_{\rm ng} \propto GM\rho R_T \Delta^{-2} \quad (5)$$

Ratios of pressures due to acceleration, self-gravitation (subscript sg), gravity-gradient (subscript gg), and a nearby mass (subscript ng) are formed with the capillary pressure, P_c:

$$B = \rho a R_T^2/\sigma \quad (6)$$

$$B_{\rm sg} = \rho^2 G R_T^3/\sigma \quad (7)$$

$$B_{\rm gg} = \rho \Omega^2 R_T^3/\sigma \quad (8)$$

$$B_{\rm ng} = \rho G M R_T^2/(\Delta^2 \sigma) \quad (9)$$

Material properties for differing propellants affect these ratios through density ρ and surface tension σ.

The magnitude of these ratios for liquid oxygen ($\rho \approx 1.2$ gm/cm and $\sigma \approx 14$ dyne/cm) in a circular Earth orbit 800 km above the surface vs tank radius are shown in Fig. 1. For the nearby body case $B_{\rm ng}$, $M = 10^5$ kg located 20 m from tank center to approximate docking of a space shuttle. When redrawn for hydrazines and nitrogen tetraoxide Fig. 1 changes little, but for liquid hydrogen the self-gravitation line moves to the right because of the ρ^2 dependence. The interaction of the effect denoted by the line and capillary effects needs to be considered for tanks larger than the value of R_T at which the line crosses above the heavy dashed line at ordinate value of 0.1. For example, when gravity gradient creates a pressure change, 10% or more of capillary pressure jump, it is likely that these two effects will interact and alter liquid positioning in the propellant tank.

The lessons from the scaling results in Fig. 1 are as follows:

1) The $B_{\rm ng}$ line in the lower right corner of the plot shows that a docking spacecraft is to be ignored. Even a larger-mass spacecraft will not increase $B_{\rm ng}$ substantially because the separation distance Δ increases as spacecraft size M increases.

2) A micro-g acceleration of the spacecraft is important for tanks when $R_T > 0.5$ m.

3) For $R_T < 3$ m, a micro-g acceleration is the largest noncapillary effect.

4) Above $R_T \approx 1.5$ m, a micro-g acceleration has a larger impact than capillary effects.

5) When not accelerating, that is, $B = 0$, gravity-gradient effects are the largest noncapillary effect when in an orbit. Gravity-gradient effects appear to be nonnegligible for $R_T > 0.8$ m.

6) For $R_T \approx 2$ m and above, self-gravitational effects need to be considered. The ratio of self-gravitation to gravity-gradient effects is independent of tank size when in orbit. However, during a coasting period in interplanetary travel, gravity gradient is negligible and self-gravitation will remain finite.

In addition, it is shown next that relying solely on the magnitudes of these effects can be misleading because an effect that eliminates symmetry can have substantial impact even at small magnitudes.

III. Example Impact

Propellant positioning in a generic vane-type propellant management device [3] in low Earth orbit including the gravity gradient is studied to illustrate how the results of the scaling exercise may impact spacecraft systems. A spherical tank with a center-post style propellant management device (PMD) vane assembly of four vanes of radial extent $0.1R_T$ as shown in Fig. 2 is modeled. The four-vane center-post PMD is common in spacecraft. Additional vane material near the ends of the straight vanes can differ substantially but does not affect the topology of the ullage bubble until very late in the mission. The 3-D static equilibrium liquid distribution in the tank and vanes is computed over a range of tank sizes and liquid fill fractions using the Surface Evolver code [4,5]. Even in uniform zero gravity, asymmetric propellant positions can exist as the low-energy static equilibrium condition [6] and if unanticipated can introduce large errors into tank thermal models. This error can even impair propellant gauging efforts [7]. The inclusion of the gravity-gradient physics alters the symmetry of the problem, and this is found to be an important effect not indicated by the relative magnitudes shown in Fig. 1.

Two topologies are examined for a zero contact angle propellant wetting the tank interior and center-vane PMD. The symmetric tank and PMD geometry can have asymmetric propellant positions as the lowest-energy state because both research [8], the Gravity Probe-B satellite [9], and previous modeling [6] have shown this to be the case. The geometries are classified by the ullage bubble being either a toroidal or spherical topology, as shown in Fig. 3. The gray surface is the liquid-vapor interface between the propellant and the pressurant gas. Shown are bubbles with topology of a torus (left) and a sphere (right). These are for tank radius of 1 m, gravity gradient included for an orbital altitude of 800 km, and liquid fill fractions of 0.5 and 0.3, respectively. The toroidal topology bubble encircles the central PMD vanes and the interior corners of the vanes are wetted by the propellant, consistent with the contact angle being less than the critical wetting angle for a 90 deg corner [10]. In perfect zero gravity,

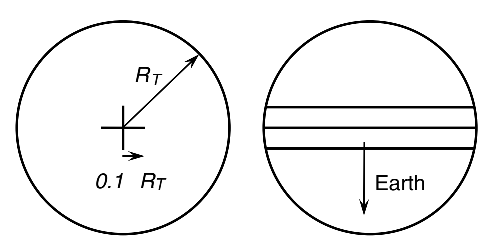

Fig. 2 Tank and PMD geometry. The central vane structure without end vanes provides a general model that captures the fluids phenomena important to general PMD design.

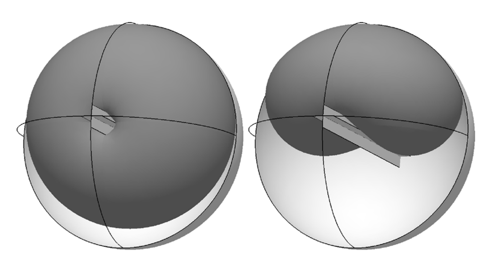

Fig. 3 Example solutions for the propellant positions. The left image is a torodial topology bubble that is located asymmetrically about the vanes due to the gravity gradient. The spherical topology bubble on the right is positioned by gravity gradient but also exists as a minimum-energy solution in some aerogravity cases [6].

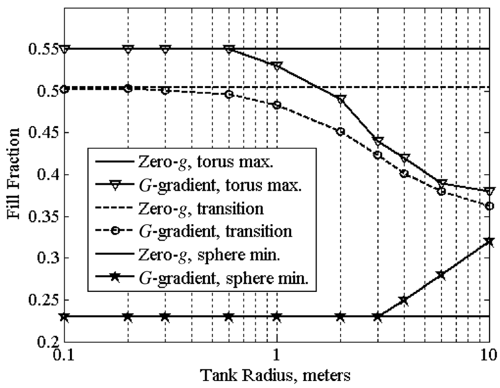

Fig. 4 Gravity-gradient impact for liquid oxygen at orbital altitude of 800 km. Results are exact in tank radius and have uncertainly in the fill fraction of ±0.01.

the toroidal bubble is found to be stable as a symmetrical distribution for fill fractions under 0.14 with this PMD. The toroidal bubble exists as a nonsymmetric solution, pushed against one wall of the tank and thinned (similar to the gravity-gradient solution shown on the left in Fig. 3) for fill fractions up to 0.55. The spherical topology bubble at the fill fractions of interest is located off center and generally impinges on either one or three vanes, as shown on the right in Fig. 3. Which of these topologies (or even perhaps other topologies) can exist for a specific fill fraction must be determined to develop accurate knowledge of propellant positioning, even for perfect zero gravity [6].

Results from computations of over 250 cases for liquid oxygen in a range of tank size and fill fractions are presented in Fig. 4. This figure present three pairs of lines: one pair for each of three phenomena, described in the following paragraphs. The two lines in each pair denote the zero-gravity result (line with no markers) and the result with gravity-gradient included (line with markers). The upper and lower pairs of lines in the figure are coincident for smaller tank radii, indicating negligible impact of the gravity-gradient effect in that region of the curve.

Consider first the pair of dashed lines that denote the liquid fill fractions above which the spherical topology solution has lower energy than the toroidal solution, and thus is the most stable solution. The straight horizontal line at $f = 0.504$ is the result for zero gravity. For zero gravity, a fill fraction $f > 0.504$ will cause the asymmetric spherical topology to be more stable than the toroidal one. This result is strictly a function of tank geometry and is regardless of liquid properties and tank radius. The dashed line marked by circles is the result when the gravity-gradient effect is included in the modeling. This shows that as tank size grows, the propellant will exist in the spherical topology solution, with off-axis mass center and nonuniform wetting of the tank wall, for more of the mission.

Note that for small radii, $R_T < 0.2$ m, the impact of gravity gradient on this transition fill fraction is small. However, for tank radii as small as $R_T \approx 0.3$ m, the gravity-gradient effect causes the asymmetric spherical topology bubble to exist as the low-energy solution for a wider range of fill fractions as R_T increases. This expanded range in which the spherical topology solution exists as the low-energy topology extends as low as $f = 0.36$ instead of $f = 0.504$ at $R_T = 10$ m. Not shown here but of interest is that for $f > 0.14$ in zero gravity the solution with a toroidal topology is an asymmetric solution contacting the tank wall for a only portion of the circumference in a plane normal to the two vanes, similar to the left-most image in Fig. 3.

The upper pair of solid lines in Fig. 4 marks the maximum fill fraction for which a toroidal topology bubble can exist. For all fill fractions above this line, the pressurant gas will form an asymmetric bubble with spherical topology. The horizontal solid line at $f = 0.55 \pm 0.01$ m is the zero-gravity result. The solid line marked with triangles is the result when gravity gradient is included. The impact of the gravity gradient begins near $R_T \approx 0.6$ m (triangles on the curve mark the ten values of the tank radius at which numerous computations were performed to identify this limit in fill fraction). It is worth noting that $R_T \approx 0.6$ m is not futuristic but is approximately a large hydrazine tank in a modern communications satellites. For larger tanks, the intrinsically asymmetrical spherical topology bubble will exist for a longer portion of the mission.

The lower pair of solid lines marks the minimum fill fraction for which the spherical topology bubble can exist in the tank and PMD. As for the other pairs of lines, the zero-gravity result is a straight horizontal line, this time at $f \approx 0.23$ m. The gravity-gradient effect (solid line marked with stars) does not become significant in this case unless it is greater than $R_T \approx 3$ m. This is likely because the energies involved in the fluid process that seeks to minimize energy are dominated here by the free surface energy and thus the gravity gradient is less influential for tank radii below 3 m. At all fill fractions below this line, such as late in the mission, the ullage bubble must form the toroidal topology. This toroidal bubble is asymmetric with gravity gradient included and is also asymmetric for zero gravity when $f > 0.14$.

IV. Conclusions

A scaling study shows that several nonuniform gravity (or acceleration) field effects could be nonnegligible for tanks with a radius on the order of 1 m. At larger sizes, nonuniform effects such as gravity gradient and self-gravitation can dominate. Gravity-gradient effects on propellant distributions in vane-type surface tension propellant management devices in low Earth orbit can be found even in tanks of a radius as small as 0.3 m. Shifts in the location of the mass center of the liquid will cause errors in the spacecraft dynamic model if the shifts are unaccounted for. Similarly, tank thermal models will contain significant errors if the substantial changes in dry and wetted regions of the tank wall are not modeled accurately. Spacecraft acceleration should have similar effects. The computational tool for such tasks, Surface Evolver, exists and is available for free. Additionally, self-gravitational effects on the liquid mass will likely become important for tank radius greater than approximately 2 m in orbit and especially during coasting periods of travel between orbits around different masses. Thus, gravity-gradient and self-gravitational effects need to be included in the design of spacecraft with tanks of diameters of 2 m, or else suboptimal capabilities and safety may be designed into the system. For example, propellant draining, thermal insulation of tanks for cryogenic propellants, and dynamic model of the spacecraft can all be in error.

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